Investigations of a Toy Model of Dark Matter Clustering

Shayla Brooks, Vincent Davis, Elizabeth City State University
Nelvin Thomas, University of the Virgin Islands
Tecora Hicks, Daniel M. Smith, Jr., South Carolina State University

The growth of dark matter clusters in the early universe is investigated by using a two-dimensional toy model developed by Krauss and Starkman. The model is implemented in the *Mathematica* software so that the variation of model parameters can be understood visually. Effects investigated include expansion, growth of the horizon, and free-streaming. Because dark matter is thought to be the seeds for the formation of galaxies, the correlation function for a set of model parameters is calculated and compared with that determined from an actual galaxy catalog.

I. Introduction

This paper describes an investigation of a toy model of dark matter clustering. Such clustering is characterized by the same statistic used to characterize the clustering of galaxies: the angular correlation function. For a random distribution, the probability of finding N galaxies in a solid angle $d\Omega$ is

$$dP = \frac{d\Omega}{4\pi}. \tag{1}$$

The probability of finding N_1 galaxies in solid angle $d\Omega_1$, and N_2 galaxies in solid angle $d\Omega_2$ for a random distribution is

$$dP_1\, dP_2 = \frac{d\Omega_1}{4\pi}\frac{d\Omega_2}{4\pi}\ . \tag{2}$$

For a clustered distribution, the angular correlation coefficient $w(q)$ is defined by

$$dP_1\, dP_2 = \frac{d\Omega_1}{4\pi}\frac{d\Omega_2}{4\pi}\ (1 + w(\vartheta)). \tag{3}$$

The angular correlation coefficient is measured in this work for a two-dimensional model featuring dark matter (adjustable from hot to cold), an expanding universe, and horizon growth. The purpose of this work is to demonstrate in a simple model how dark matter forms the seed for galaxy clustering. In Section II, we give a basic description of galaxy clusters. In Section III, we described how dark matter affects the formation of galaxy clusters. These sections introduce the physics of the toy model described in Section IV, with results described in Section V. Section VI summarizes this work.

II. Galaxy Clusters

A large aggregation of one hundred billion or more stars constitutes a galaxy. Because the distances between most galaxies are far greater than their sizes, the entire effect of the gravity of billions of stars is treated as being equivalent to a single mass in the toy model.

Galaxies themselves aggregate into groups of a few tens of objects. For example, our own galaxy, the Milky Way, is a member of a group called the Local Group. The Local Group consists of about thirty galaxies, in a region of size 1.3 Mpc. The next largest aggregation of galaxies is called a cluster. Its size is a few Mpc, and it may contain from a few hundred to a thousand members.

Galaxy clusters can form even larger structures, called superclusters. Superclusters range in size from about 20 to 50 Mpc across. Astronomers have amassed evidence that the Milky Way is part of an enormous flattened supercluster called the Local Supercluster. This Supercluster contains the Local Group, the Virgo, the Coma clusters, and about 100 other clusters.

Cluster growth is simulated in the toy model by including the effect of horizon growth, explained in Section IV. Evidence is strong that galaxies were clustered at their births; this clustering is thought to be seeded by dark matter.

III. Dark Matter

Structure growth in the universe, including galactic clustering, is thought to be due to density fluctuations in dark matter. The dark matter is speculated to be an undiscovered elementary particle which can be either “hot,” or “cold.” These two kinds of dark matter lead to very different forms of density fluctuations in the universe and, consequently, to very different structures in the universe.

Hot dark matter tends to have few density fluctuations associated on small scales. Since dark matter is assumed to be electrically neutral, the key characteristic of this elementary particle will be its mass. As the universe cools, the dark matter particles effectively decouple from the rest of matter, leaving the dark matter to move at very high speeds. The adjective, hot, merely means that the dark matter particles are moving at very high speeds after decoupling.

On the other hand, if the dark matter particles are heavier than the familiar elementary particles, the situation will be reversed. The proton has a mass of about one GeV. If the dark matter particle has a mass fifty times greater, it would move at a very low speed compared to protons in the early universe. Therefore, it would have a very low speed when it decoupled from the rest of the universe, hence the name cold dark matter.

IV. Clustering Model

We consider a two-dimensional model of the universe containing only galaxies.[1] The model is studied to see that it generates the previously described effects, and to determine if it generates a correlation coefficient similar to that of actual galaxies.

The distribution of galaxies generated by the clustering model is compared to the same number of randomly distributed galaxies. For both the random and clustered distributions, the number of distinct pairs within a solid angular region is n(n-1)/2. The number of distinct pairs generated by the clustering model is represented by <DD>, and the number of distinct pairs for random galaxy distribution is <RR>. The two point angular correlation is

$$w(\theta) = \frac{\langle DD \rangle}{\langle RR \rangle} - 1 . \qquad (4)$$

The angular correlation is also calculated by using the method suggested by equation (3), namely counting the pairs formed by galaxies in different angular regions.

Gravity is modeled, not by using the inverse-square law, but by considering nearest-neighbor galaxies to be under each other’s gravitational influence. The model implements the expanding universe by doubling the number of rows and columns with each iteration. The horizon growth is the clustering radius and it expands as the universe expands. In other words, with each iteration, the horizon size increases, causing the effective gravitational range to expand.

The model includes free streaming, a property of dark matter which is essentially a measure of its kinetic energy. As the free streaming length increases, the dark matter becomes hot and clumps less on small scales. Conversely, if the free streaming length decreases, the dark matter becomes cold and there is more clumping on small scales.

V. Results

Figures 1 and 2 show results from the model for free-streaming lengths (in dimensionless units) 1 and 8. The plots show that the clustering is more pronounced on the smaller length scale for the smaller free-streaming length,

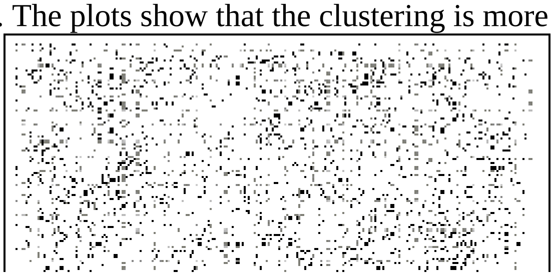
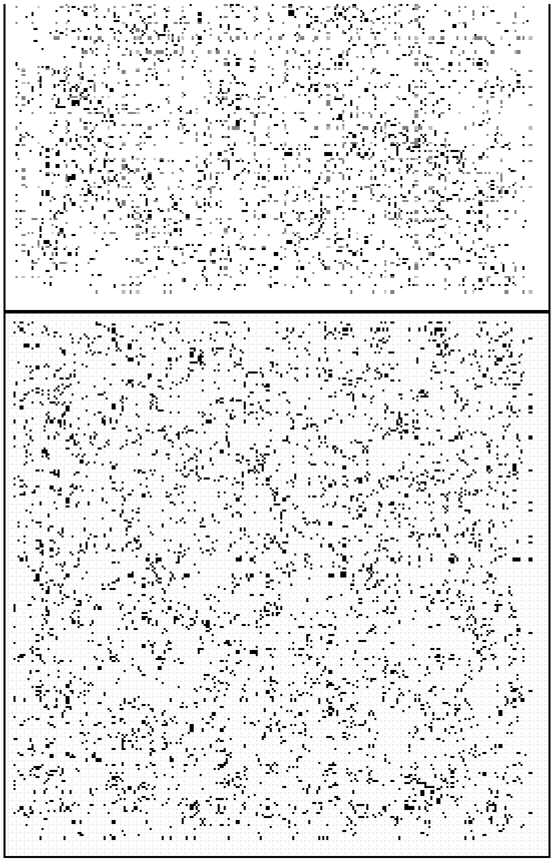


Fig. 2. Hot dark matter (free streaming parameter equals 8).

Fig. 1. Cold dark matter (free streaming parameter equals 1).



corresponding to cold dark matter. The larger free-streaming length corresponds to hot dark matter.

Plots of angular correlation versus angle using the first method of Section IV (Figures 3 and 4) do not distinguish between the density plots, although these plots do correspond well to the angular correlation measurements of actual galaxies.[2] Note, for example, that pairing is less likely between distant galaxies as reflected by the decrease in the angular correlation. The second method of finding the angular correlation noted in Section IV is better at showing that clumping is enhanced for cold dark matter (Figure 5).

Both techniques suffer from insensitivity to the differences between cold and hot dark matter because of fluctuations in the number of points in a “bin.”

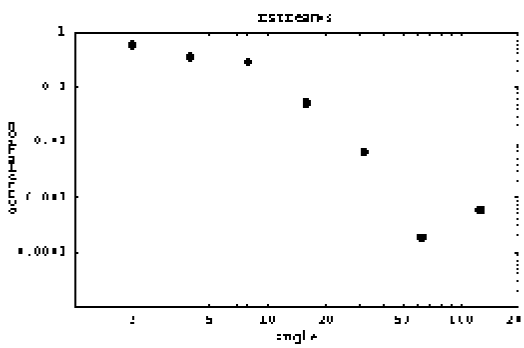


Fig. 3. Angular correlation for cold dark matter (arbitrary angle units).

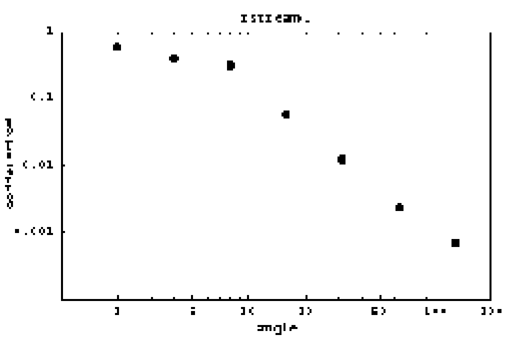


Fig. 4. Angular correlation for hot dark matter (arbitrary angle units).

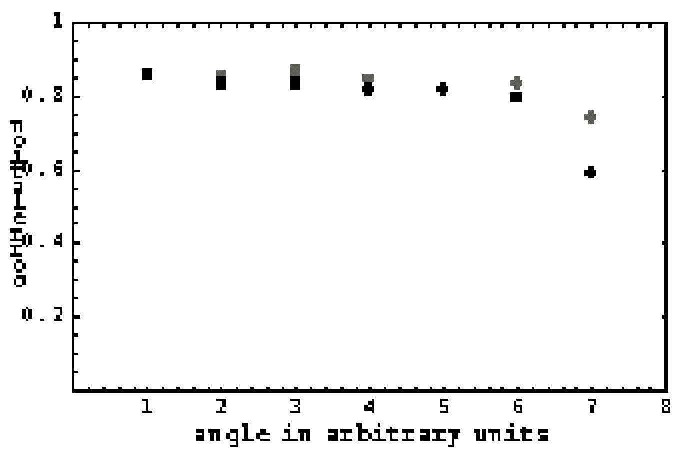


Fig. 5. Cold dark matter (gray points) shows enhanced clustering compared to hot dark matter (black points).

Figures 6 and 7 show that a more promising method of treating the problem of finding the angular correlation is to take a frequency spectrum of the data. Note for example that the histogram for cold dark matter is more sharply peaked than that for hot dark matter.

VI. Conclusions

The Krauss-Starkman toy model is successful at reproducing in density plots the effects central to dark matter clustering, including gravitational attraction, expansion of the universe, horizon growth and free-streaming. An angular correlation function is calculated which marginally distinguishes between cold and hot dark matter. Preliminary results indicate that finding frequency spectra is a more promising method of analysis.

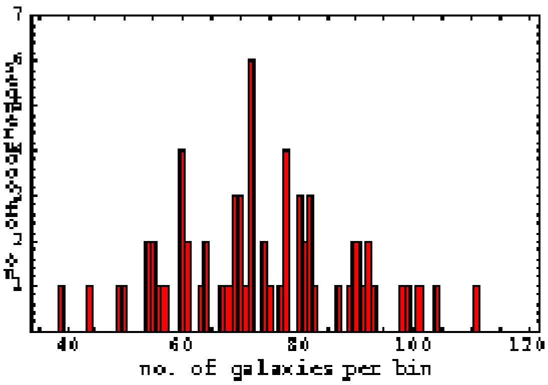


Fig. 6. Frequency spectrum for cold dark matter.

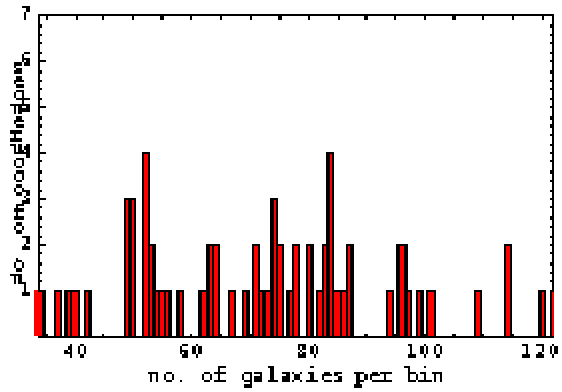


Fig. 7. Frequency spectrum for hot dark matter.

VII.

Acknowledgments

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VIII. References

[1] Lawrence M. Krauss and Glenn D. Starkman, *Teaching About Cosmology: A Programmed Guide for High School Teaching and Learning*. (American Association of Physics Teachers College Park, MD, 1999).

[2] Edward J. Groth and P. J. E. Peebles, “Statistical Analysis of Catalogs of Extragalactic Objects. VII. Two- and Three-Point Correlation Functions for the High-Resolution Shane-Wirtanen Catalog of Galaxies,” *The Astrophysical Journal* **217**, 385(1977).